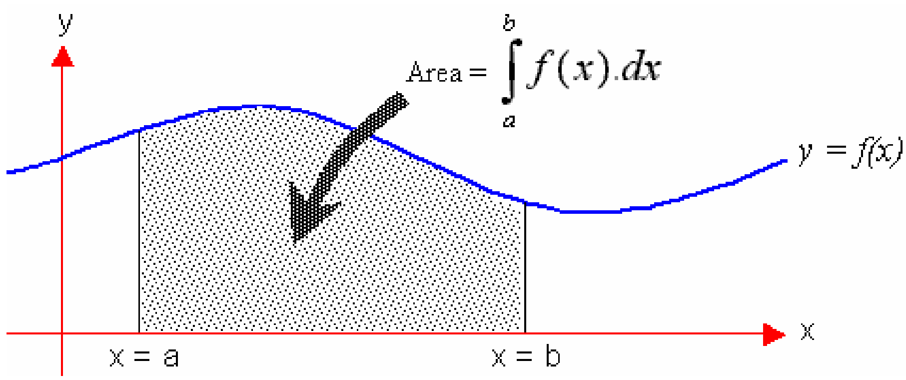
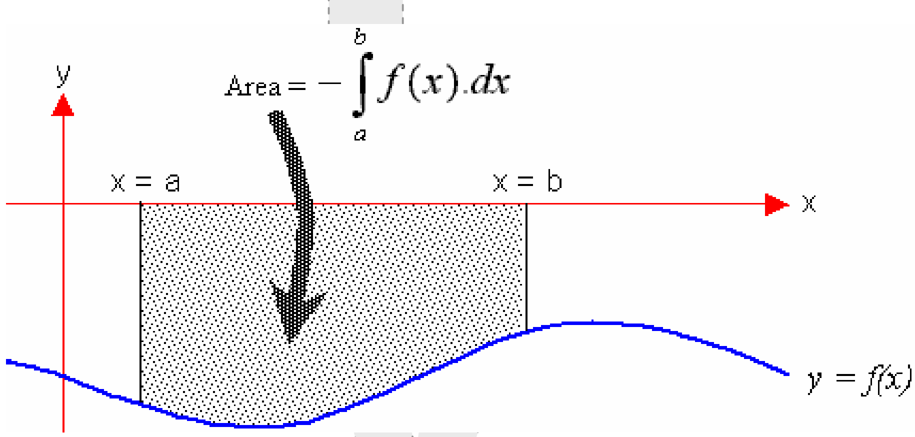
AREA UNDER CURVES

The definite integral can be used to find the area between a graph curve and the ‘x’ axis, between two given ‘x’ values. This area is called the ‘area under the curve’ regardless of whether it is above or below the ‘x’ axis. When the curve is above the ‘x’ axis, the area will be the definite integral of function f(x)

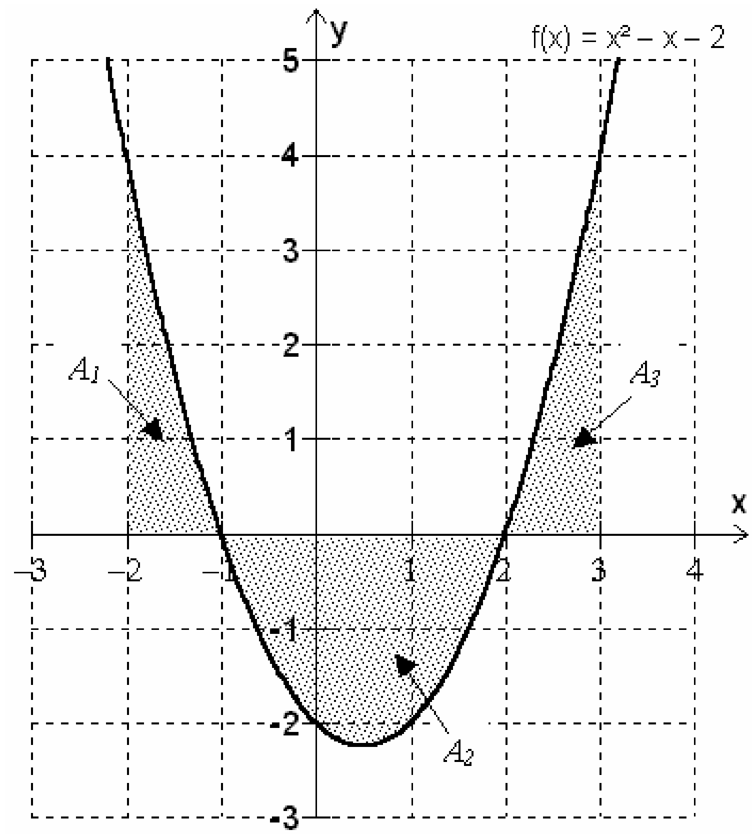


but when the graph line is below the ‘x’ axis, the value of definite integral will be negative so the area will be given by:



The total area can be founded by adding different parts like given below :

For example, to find the area between the graph of: $y = x^2 - x - 2$ and the 'x' axis, from $x = -2$ to $x = 3$, we need to calculate three separate integrals:



The zeros of the function f(x) that lie between -2 and 3 form the boundaries of the separate area segments.

In this case there are zeros at $x = -1$ and $x = 2$ (see graph above) and so three separate areas must be found: A_1, A_2 and A_3 as follows:

$$A_1 = \int_{-2}^{-1} (x^2 - x - 2)\, dx$$

$$A_2 = \int_{-1}^{2} (x^2 - x - 2)\, dx$$

$$A_3 = \int_{2}^{3} (x^2 - x - 2)\, dx$$

So the total area of the shaded region between the function and the graph from $x = -2$ to $x = 3$ is given by:

$$A = A_1 + A_2 + A_3$$

CURVE TRACING

The following outline procedure is to be applied in Sketching the graph of function y = f(x) which in turn will be extremely useful to quickly and correctly evaluate the area under the curves.

(A) **Symmetry:** The symmetry of the curve is judged as follows :

- (i) If all the powers of y in the equation are even then the curve is symmetrical about the axis of x.
- (ii) If all the powers of x are even, the curve is symmetrical about the axis of y.
- (iii) If powers of x & y both are even, the curve is symmetrical about the axis of x as well as y.
- (iv) If the equation of the curve remains unchanged on interchanging x and y, then the curve is symmetrical about y = x.
- (v) If on interchanging the signs of x & y both the equation of the curve is unaltered then there is symmetrical about y = x.

(B) Find dy/dx & equate it to zero to find the points on the curve where you have horizontal tangents.

(C) Find the points where the curve crosses the x-axis & also the y-axis.

(D) Examine if possible the intervals when f(x) is increasing or decreasing. Examine what happens to 'y' when $x \to \infty$ or $-\infty$.

AREA OF BOUNDED REGIONS

THEOREM Let f (x) be a continuous function defined on [a, b]. Then, the area bounded by the curve y = f (x), the x-axis and the ordinates x = a and x = b is given by

$$\int_a^b f(x)\,dx \text{ or } \int_a^b y\,dx.$$

USEFUL RESULTS

(i) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is π **ab**.

(ii) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is $\frac{16ab}{3}$.

(ii) Area included between the parabola $y^2 = 4ax$ & the line y = mx is $\frac{8a^2}{3}m^3$.

(iv) The area of the region bounded by $y^2 = 4ax$ and its latus rectum is $\frac{8a^2}{3}$ sq. units.

(v) The area of the region bounded by one arch of sin (ax) and x-axis is or cos (ax) is $\frac{2}{a}$ sq. units.